Consequences of vacuum polarization on electromagnetic waves in a Lorentz-symmetry breaking scenario

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Abstract

The propagation of electromagnetic waves in a Lorentz-symmetry violating scenario is investigated in connection with non-linear (photon self-interacting) terms induced by quantum effects. It turns out that the photon field acquires an interesting polarization state and, from our calculations of phase and group velocities, we contemplate different scenarios with physically realizable magnetic fields and identify situations where non-linearity effects dominate over Lorentz-symmetry breaking ones and vice-versa.

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1 Introduction

In the endeavor of unveiling the physics that unfold beyond the Standard Model, our best efforts have led us to a whole class of CPT- and Lorentz-symmetry breaking models. The most suitable framework to deal with questions within these scenarios is the effective theory approach referred to as the Standard Model Extension (SME) [1], where it is possible to embody spontaneous Lorentz-symmetry breaking and still keep desired properties of standard Quantum Field Theory. Recently, a great deal of efforts have been devoted to the search of measurable consequences of this kind of breaking [2,3]. On the other hand, although direct measurements of nonlinear effects of electromagnetic processes in vacuum remain elusive, intensive experimental research is still in course; for example, in the PVLAS apparatus [4], now operating in its Phase II [5,6]. Also, we point out another interesting class of experiments (light-shining-through-a-wall, LSW) where non-linear effects (photon-photon-ALP coupling) are to be detected [7,8].

Here, we address the issue of light propagation in a non-trivial background. We consider the vacuum in the presence of a strong magnetic field in a scenario with Lorentz-symmetry breaking. Non-linearity due to quantum field-theoretic effects, on the one hand, and Lorentz-symmetry violation induced phenomena, on the other hand, are all very tiny at our accessible energy scales. But, precision measurements in electromagnetic processes can be carried out to set up constraints and bounds on physical parameters related to these non-linear and Lorentz-symmetry violating effects, shedding some light on a physics that becomes very significant at high energies, at scales close to the threshold for more fundamental theories.

Our work sets out to propose a discussion on the interplay between non-linear and Lorentz-symmetry violation considered contemporarily. We have clear that both effects are per sé very small and we actually understand that their interference is much far from our precision measurements. However, we are able to show how these phenomena add to each other and we identify physical situations where they can be of the same order. Also, our results are shown to be compatible with current bounds carefully analyzed in the work of Refs. [3,9] and papers quoted therein. In the present work, by virtue of the Lorentz-symmetry violation in the presence of a strong magnetic field in vacuum, we find out that the photon acquires a peculiar longitudinal polarization that can be tested at laboratory scales, possibly in a PVLAS-like configuration, for instance. Also, the characteristic phase and group velocities, whose difference is useful to characterize dispersion, are worked out. Along the paper, we shall adopt the metric $\eta^{\mu\nu} = (+,-,-,-)$ and the Levi-Civita tensor defined such that $\epsilon^{0123} = 1$.

2 Model under consideration

The model under consideration is described by the Lagrangian density:

$$\mathcal{L} = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{R}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{S}{8\pi}(\mathbf{E}\cdot\mathbf{B})^2 + \frac{1}{16\pi}\epsilon^{\mu\nu\lambda\rho}A_\nu(\hat{k}_{AF})_\mu F_{\lambda\rho} \tag{1}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, A^μ is the vector potential, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the field strength and $(\hat{k}_{AF})_\mu$ is the vector operator that characterizes the Lorentz-violating Chern-Simons term proposed in the work of Ref. [10],

$$\begin{aligned}(\hat{k}_{AF})_\mu &= \sum_{d=3,5,\ldots}(k_{AF}^{(d)})_\mu^{\alpha_1,\alpha_2,\ldots,\alpha_{(d-3)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-3)}} \\ &= (k_{AF}^{(3)})_\mu + (k_{AF}^{(5)})_\mu^{\alpha_1,\alpha_2}\partial_{\alpha_1}\partial_{\alpha_2} + (k_{AF}^{(7)})_\mu^{\alpha_1,\alpha_2,\alpha_3,\alpha_4}\partial_{\alpha_1}\partial_{\alpha_2}\partial_{\alpha_3}\partial_{\alpha_4} + \ldots\end{aligned} \tag{2}$$

It is important to notice that the last term in (1) along with the expression of Eq. (2) generalizes the so-called Carroll-Field-Jackiw term [2], for which $(k_{AF}^{(3)})^\mu = v^\mu$ and $(k_{AF}^{(d)})^\mu = 0$ for $d \geq 5$.

Taking

$$R = \frac{e^4}{45\pi m_e^4}\ ,\ S = 7R\ , \tag{3}$$

where m_e is the electron mass, and, for $(\hat{k}_{AF})_\mu = 0$, we are led to the well-known Euler-Heisenberg Lagrangian [11]. This describes an effective model up to first order in the parameters R and S; so, our calculations shall be carried out only up to first order in R and S. We stress that we discard terms of order Rv^μ and Sv^μ, since v^μ is very tiny according to the available experimental data [3].We also point out that our (Lorentz- symmetry violating) Chern-Simons term in the Lagrangian (1) replaces the P- and CP-violating term, $(\mathbf{B}^2 - \mathbf{E}^2)\,\mathbf{E}\cdot\mathbf{B}$, considered by Hu and Liao in the paper of Ref. [12]. We adopt the choice of the Chern-Simons term to describe CPT violation and, then, to study its consequence in presence of the non-linear $(\mathbf{B}^2 - \mathbf{E}^2)$ and $(\mathbf{E}\cdot\mathbf{B})^2$ terms.

From now on, we shall always restric to field configurations where the effects imposed by the non-linear terms as well as the ones imposed the Chern-Simons-like term in (1) are small perturbations.

In order to bring the dynamical equations into a compact and convenient form,

we shall define the vectors $\mathbf{D}$ and $\mathbf{H}$, in analogy to the electric displacement and magnetic field strength, as follows:

$$\mathbf{D} = 4\pi \frac{\partial}{\partial \mathbf{E}} \left[\frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{R}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{S}{8\pi}(\mathbf{E}\cdot\mathbf{B})^2 \right] ,$$

$$\mathbf{H} = -4\pi \frac{\partial}{\partial \mathbf{B}} \left[\frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{R}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{S}{8\pi}(\mathbf{E}\cdot\mathbf{B})^2 \right] , \tag{4}$$

so that,

$$\mathbf{D} = \mathbf{E} + 2R(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{E} + S(\mathbf{E}\cdot\mathbf{B})\mathbf{B} ,$$

$$\mathbf{H} = \mathbf{B} + 2R(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{B} - S(\mathbf{E}\cdot\mathbf{B})\mathbf{E} . \tag{5}$$

The usual methods used to study light propagation in a non-trivial vacuum, due to the presence of external fields or boundary conditions, can be applied here [13,14,15,16,17,18,19,20].

From now on, we shall be considering the propagation of electromagnetic waves in the presence of an external constant and uniform magnetic field. The goal is to identify the polarization states of the photon field in this situation. To this end, let us denote the electric and magnetic fields of the propagating wave by $\mathbf{E}_P$ and $\mathbf{B}_P$ respectively, and the external magnetic field by $\mathbf{B}_0$.

The dynamical equations that stem from the Lagrangian density (1) read as below:

$$\nabla \cdot \mathbf{D} = \hat{\mathbf{k}}_{AF} \cdot \mathbf{B} \quad , \quad \nabla \cdot \mathbf{B} = 0 ,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} ,$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + (\hat{k}_{AF})^0 \mathbf{B} - (\hat{\mathbf{k}}_{AF}) \times \mathbf{E} . \tag{6}$$

The simplest solutions, written in terms of the propagating and external fields, are of the form:

$$\mathbf{E} = \mathbf{E}_P + \delta\mathbf{E} \quad , \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_P + \delta\mathbf{B} , \tag{7}$$

where $\delta\mathbf{E}$ and $\delta\mathbf{B}$ stand for the contributions that do not propagate, as dictated by the set of equations (6).

In view of what has been exposed above, the following conditions may be adopted:

$$
\begin{aligned}
|\delta \mathbf{E}| << |\mathbf{E}_P| << (R)^{-1/2}, (S)^{-1/2}, \\
|\delta \mathbf{B}| << |\mathbf{B}_P| << |\mathbf{B}_0| << (R)^{-1/2}, (S)^{-1/2} \ .
\end{aligned}
\tag{8}
$$

Substituting (7) into (5), taking into account the relations (8) and discarding terms of order $R\delta\mathbf{E}$, $S\delta\mathbf{E}$, $R\delta\mathbf{B}$, $S\delta\mathbf{B}$, $\mathbf{E}_P^2$, $\mathbf{B}_P^2$ and $\mathbf{E}_P \cdot \mathbf{B}_P$, we can write that

$$
\begin{aligned}
\mathbf{D} = \mathbf{E}_P + \delta\mathbf{E} - 2RB_0^2\mathbf{E}_P + S(\mathbf{E}_p \cdot \mathbf{B}_0)\mathbf{B}_0 \ , \\
\mathbf{H} = \mathbf{B}_0 + \mathbf{B}_P + \delta\mathbf{B} - 2RB_0^2\mathbf{B}_P - 4R(\mathbf{B}_0 \cdot \mathbf{B}_P)\mathbf{B}_0 - 2RB_0^2\mathbf{B}_0 \ .
\end{aligned}
\tag{9}
$$

From this, it can be readily seen that the displacement field and the magnetic field strength for the propagating fields can be expressed by

$$
\begin{aligned}
\mathbf{D}_P &= \mathbf{E}_P - 2RB_0^2\mathbf{E}_P + S(\mathbf{B}_0 \cdot \mathbf{E}_P)\mathbf{B}_0 \ , \\
\mathbf{H}_P &= \mathbf{B}_P - 2RB_0^2\mathbf{B}_P - 4R(\mathbf{B}_0 \cdot \mathbf{B}_P)\mathbf{B}_0 \ .
\end{aligned}
\tag{10}
$$

Replacing (7) and (9) into (6) yields two sets of equations, one for the non-propagating fields and the another for the propagating ones, whose dynamics we are interested in. We then have:

$$
\begin{aligned}
\nabla \cdot \mathbf{D}_P = \hat{\mathbf{k}}_{AF} \cdot \mathbf{B}_P \quad , \quad \nabla \cdot \mathbf{B}_P = 0 \ , \\
\nabla \times \mathbf{E}_P = -\frac{\partial \mathbf{B}_P}{\partial t} \ , \\
\nabla \times \mathbf{H}_P = \frac{\partial \mathbf{D}_P}{\partial t} + (\hat{k}_{AF})^0 \ \mathbf{B}_P - (\hat{\mathbf{k}}_{AF}) \times \mathbf{E}_P \ .
\end{aligned}
\tag{11}
$$

The explicit form of the Euclidean tensors for electric permittivity and inverse magnetic permeability in these equations can be obtained from (10),

$$
\begin{aligned}
\varepsilon_P^{ij} &= (1 - 2RB_0^2)\delta^{ij} + SB_0^iB_0^j \ , \\
(\mu_P^{-1})^{ij} &= (1 - 2RB_0^2)\delta^{ij} - 4RB_0^iB_0^j \ ,
\end{aligned}
\tag{12}
$$

where δ^{ij} stands for the Kronecker delta. Therefore, in components, Eqs (10) reads as below:

$$
D_P^i = \sum_{j=1}^{3} \varepsilon_P^{ij} E_P^j \quad , \quad H_P^i = \sum_{j=1}^{3} (\mu_P^{-1})^{ij} B_P^j \ .
\tag{13}
$$

As usual, let us search for plane waves configurations for the propagating fields, $\mathbf{E}_P$ and $\mathbf{B}_P$, in the form

$$\mathbf{E}_P = \mathbf{e} \ \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)] \ ,$$
$$\mathbf{B}_P = \mathbf{b} \ \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)] \ , \qquad (14)$$

where $\mathbf{e}$ and $\mathbf{b}$ are constant and uniform vectors. For simplicity, we restrict our attention to the case where the wave vector $\mathbf{k}$ is perpendicular to the external magnetic field, $\mathbf{B}_0$, i.e. $\mathbf{k}\cdot\mathbf{B}_0 = 0$, and the coordinate system will be taken in such a way that

$$\mathbf{B}_0 = B_0\hat{x} \quad , \quad \mathbf{k} = k\hat{z} \ . \qquad (15)$$

In such a system, the tensors in (12) can be brought into the form

$$\varepsilon_P^{ij} = (1 - 2RB_0^2)\delta^{ij} + SB_0^2\delta^{i1}\delta^{j1} \ ,$$
$$(\mu_P^{-1})^{ij} = (1 - 2RB_0^2)\delta^{ij} - 4RB_0^2\delta^{i1}\delta^{j1} \ , \qquad (16)$$

and the differential operators $(\hat{k}_{AF})^0$ and $(\hat{\mathbf{k}}_{AF})$ operating on the fields (14) can be substituted by the vectors

$$(\hat{k}_{AF})^0 \to (k_{AF})^0 = \sum_{d=3,5,\ldots} (k_{AF}^{(d)})^0 \ {}_{\alpha_1,\alpha_2,\ldots,\alpha_{(d-3)}} k^{\alpha_1}\ldots k^{\alpha_{(d-3)}}$$
$$(\hat{\mathbf{k}}_{AF}^{(d)}) \to (\mathbf{k}_{AF}) = \sum_{d=3,5,\ldots} (\mathbf{k}_{AF}^{(d)})_{\alpha_1,\alpha_2,\ldots,\alpha_{(d-3)}} k^{\alpha_1}\ldots k^{\alpha_{(d-3)}} \ , \qquad (17)$$

where the indexes α_i can assume the values 0 or 3, once our wave vector is taken to be given by $k^\mu = (\omega, 0, 0, k)$.

The Lorentz-breaking term as well as the non-linear terms modify the dispersion relation. In spite of this fact, we can use the Maxwell dispersion relation, where $\omega = k$, and set $k^{\alpha_i} = k^0 = k^3 = \omega = k$ in (17) because the coefficients $(k_{AF}^{(d)})^0{}_{\alpha_1,\alpha_2,\ldots,\alpha_{(d-3)}}$ are small quantities and the wave vector k^μ is not large [6]. So, we can write expression (17) in the form

$$(\hat{k}_{AF})^0 \to (k_{AF})^0 = \sum_{d=3,5,\ldots} (k_{AF}^{(d)})^0{}_{\alpha_1,\alpha_2,\ldots,\alpha_{(d-3)}} k^{\alpha_1}\ldots k^{\alpha_{(d-3)}}$$

[6] If the wave vector becomes large, the Lagrangian (1) must include terms in higher orders in the electromagnetic fields

$$
\begin{aligned}
&= \sum_{d=3,5,...} (k_{AF}^{(d)})^0{}_{\alpha_1,\alpha_2,...,\alpha_{(d-3)}} t^{\alpha_1} t^{\alpha_2} ... t^{\alpha_{(d-3)}} |\mathbf{k}|^{d-3} \\
(\hat{\mathbf{k}}_{AF}^{(d)}) \to (\mathbf{k}_{AF}) &= \sum_{d=3,5,...} (\mathbf{k}_{AF}^{(d)})_{\alpha_1,\alpha_2,...,\alpha_{(d-3)}} k^{\alpha_1} ... k^{\alpha_{(d-3)}} \\
&= \sum_{d=3,5,...} (\mathbf{k}_{AF}^{(d)})_{\alpha_1,\alpha_2,...,\alpha_{(d-3)}} t^{\alpha_1} t^{\alpha_2} ... t^{\alpha_{(d-3)}} |\mathbf{k}|^{d-3} \qquad (18)
\end{aligned}
$$

where we have defined $t^{\alpha_i} = (1 - \delta_1{}^{\alpha_i})(1 - \delta_2{}^{\alpha_i})$.

For future convenience, we also define $(k_{AF})^\mu = \Big((k_{AF})^0, (\hat{\mathbf{k}}_{AF}^{(d)}) \Big)$.

By combining (13), (14), (16) and (18) and using then in eqs.(11), we are led to:

$$
\begin{aligned}
\sum_{i,j=1}^{3} i k^i \varepsilon_P^{ij} e^j &= (\mathbf{k}_{AF}) \cdot \mathbf{b} \ , \\
i\mathbf{k} \cdot \mathbf{b} &= 0 \ , \\
i\mathbf{k} \times \mathbf{e} &= i\omega \mathbf{b} \ , \\
\sum_{j,k,\ell=1}^{3} i\epsilon^{ijk} k^j (\mu_P^{-1})^{k\ell} b^\ell &= \sum_{j=1}^{3} -i\omega \varepsilon_P^{ij} e^j + (k_{AF})^0 b^i - \sum_{j,k=1}^{3} \epsilon^{ijk} (k_{AF})^j e^k \ , \qquad (19)
\end{aligned}
$$

where ϵ^{ijk} is the Levi-Civita 3-tensor with, $\epsilon^{123} = 1$.

From the first and second equations in (19), we conclude that

$$
e^3 = \frac{(\mathbf{k}_{AF}) \cdot \mathbf{b}}{ik} \quad , \quad b^3 = 0 \ , \qquad (20)
$$

where terms of order $R|(\mathbf{k}_{AF})|$ were discarded. These results show that the propagating magnetic field is perpendicular to the direction of the propagating waves, but it does not necessarily happen to be orthogonal to the propagating electric field; the latter actually develops a component along $(\mathbf{k}_{AF})$. This effect is strictly due to the Lorentz- symmetry breaking term in Lagrangian (1).

It remains to be found an explicit form for the propagating magnetic field as a function of $(k_{AF})^\mu$. Using the fact that $\mathbf{k} = k\hat{z}$, the third Eq. (19) leads to

$$
\mathbf{b} = \frac{k}{\omega} \hat{z} \times \mathbf{e} \ . \qquad (21)
$$

Substituting this relation into the first Eq.(20), we have

$$e^3 = \frac{1}{i\omega}(\mathbf{k}_{AF}) \cdot (\hat{z} \times \mathbf{e}) \ \rightarrow \ e^3 = \frac{1}{i\omega}\Big((k_{AF})^2 e^1 - (k_{AF})^1 e^2\Big) \ . \qquad (22)$$

From Eqs (16), the last Eq. (19), the second Eq. (20), Eqs (21) and (22), and by neglecting terms of order $S(k_{AF})^\mu$ and $R(k_{AF})^\mu$, we get a rather simple system,

$$\begin{aligned} &[(\omega^2 - \mathbf{k}^2) + kSB_0^2]e^1 - ik\Big((k_{AF})^0 - (k_{AF})^3\Big)e^2 = 0, \\ &ik\Big((k_{AF})^0 - (k_{AF})^3\Big)e^1 + [(\omega^2 - \mathbf{k}^2) + k(4RB_0^2)]e^2 = 0 \ . \end{aligned} \qquad (23)$$

Obviously, the system above has non-trivial solutions if, and only if, the determinant of the corresponding matrix vanishes. For a given wavelength, λ, the modulus of the wave vector, $k = 2\pi/\lambda$, is determined and the condition of vanishing determinant for the coefficients matrix in (23), together with relation (3), yields the frequencies below:

$$\omega_\pm = k\left[1 - \frac{1}{4}\left(11RB_0^2 \pm \sqrt{9R^2B_0^4 + \frac{4\Big((k_{AF})^0 - (k_{AF})^3\Big)^2}{\mathbf{k}^2}}\right)\right], \qquad (24)$$

from which the corresponding phase and group velocities follow:

$$V_\pm = \frac{\omega_\pm}{k} = 1 - \frac{1}{4}\left(11RB_0^2 \pm \sqrt{9R^2B_0^4 + \frac{4\Big((k_{AF})^0 - (k_{AF})^3\Big)^2}{\mathbf{k}^2}}\right) \ , \qquad (25)$$

$$\begin{aligned} V_{g\pm} = \frac{d\omega_\pm}{dk} = V_\pm + \frac{\Big((k_{AF})^0 - (k_{AF})^3\Big)^2}{|\mathbf{k}|^3} \frac{1}{\sqrt{9R^2B_0^4 + \frac{4[(k_{AF})^0 - (k_{AF})^3]^2}{\mathbf{k}^2}}} \Bigg[\pm 1 \\ \mp \frac{|\mathbf{k}|}{[(k_{AF})^0 - (k_{AF})^3]} \sum_{d=5,7,...} t^{\alpha_1} t^{\alpha_2} ... t^{\alpha_{(d-3)}} (d-3)|\mathbf{k}|^{d-4} \\ \times \Big[(k_{AF}^{(d)})^0{}_{\alpha_1,\alpha_2,...,\alpha_{(d-3)}} - (k_{AF}^{(d)})^3{}_{\alpha_1,\alpha_2,...,\alpha_{(d-3)}} \Big] \Bigg] \ . \end{aligned} \qquad (26)$$

With the results (24), (25) and (26), we are ready to carry out numerical estimates for the corrections induced by the non-linearity and the Lorentz-

symmetry breaking parameters; the latter here show up in the combination $[(k_{AF})^0 - (k_{AF})^3]$.

Let us start by taking $(k_{AF}^{(d)})_\mu = 0$ for $d > 3$ in (2) and make some estimates and draw some conclusions in this case. As stated in reference [10], in this situation the model (2) reduces to the well known Field-Jackiw model. Also, the second and third lines of (26) disappears. With the present typical values for the $(k_{AF}^{(3)})^{0^\mu}$-components ($\lesssim 10^{-43} GeV$) [9] and for intergalactic magnetic fields ($\sim 10^{-9} T \sim 10^{-18} MeV^2$), non-linear and $(k_{AF}^{(3)})$-Lorentz-symmetry breaking corrections are of a comparable order of magnitude for wavelengths in the γ - ray region of the spectrum ($\lambda \sim 10^{-13}$ m). However, in this situation, both effects are not of a measurable size, since $RB_0^2 \sim 10^{-41}$ (RB_0^2 is dimensionless). But, if we consider $(k_{AF}^{(5)})$-effects, and also consider the bound $|(k_{AF}^{(5)})| \leq 10^{-32} Gev^{-1}$ [10], Lorentz-symmetry breaking effects are still very tiny, but they dominate. Actually, based on Eqs. (24)-(26), we estimate the $(k_{AF}^{(5)})$-effects to be of order of 10^{-16}. So, in this situation, we may have Lorentz breaking effects stronger than non-linearity corrections to the group velocities.

By considering another range of magnetic fields ($\sim 10^7 T \sim 10^{-2}\ MeV^2$), non-linearity corrections (RB_0^2) are of the order of 10^{-9}. In this case, the Lorentz-symmetry breaking effects are, however, much smaller ($\sim 10^{-40}$ in the γ-ray region, and 10^{-27} in the microwave region) and then, practically undetectable. From our study, we understand that in the range of physically realizable magnetic fields (the ones produced in quark-gluon plasmas, by neutron stars or by magnetars), effects of nonlinearity, whenever detectable, are strongly dominating over the $(k_{AF}^{(3)})$-Lorentz-symmetry breaking corrections all over the electromagnetic spectrum. On the other hand, we have also identified a situation where $(k_{AF}^{(5)})$-effects may dominate over non-linearity as mentioned above.

From the results (24), (25) and (26), we also understand that detectable corrections induced by the breaking of Lorentz covariance would be present at very high energy scales, whenever string effects trigger the violation of relativistic invariance and the violating parameters may acquire values of the order of $10^{18} GeV$. However, at this regime, a new physics is at work and the Lorentz-symmetry breaking may induce other corrections to Maxwell equations which we are not contemplated here.

It can also be seen from the system (23) that each mode has only a single degree of freedom. By choosing $e_\pm^1$ as the free variable, system (23) can be solved with (24) and using (22). Defining $\Delta_\pm = \omega_\pm^2 - k^2$, the modes $\mathbf{e}_\pm = (e_\pm^1, e_\pm^2, e_\pm^3)$ for the electric field turn out to be

$$
\begin{aligned}
e^2_\pm &= e^1_\pm \ \frac{\Delta_\pm + \mathbf{k}^2 S B_0^2}{ik[(k_{AF})^0 - (k_{AF})^3]} \ , \\
e^3_\pm &= e^1_\pm \ \frac{1}{ik}\left((k_{AF})^2 - (k_{AF})^2 \frac{\Delta_\pm + \mathbf{k}^2 S B_0^2}{ik[(k_{AF})^2 - (k_{AF})^1]} \right) \ .
\end{aligned} \tag{27}
$$

The $e^3_\pm$-components are exclusively due to the breaking of Lorentz symmetry, but they are not independent, for they are proportional to the $e^1_\pm$-components. The propagating magnetic fields are given by (21), (24) and (27) as,

$$
\mathbf{b}_\pm = \left[1 + \frac{1}{4}\left(11 R B_0^2 \pm \sqrt{9 R^2 B_0^4 + \frac{4\Big((k_{AF})^0 - (k_{AF})^3 \Big)^2}{\mathbf{k}^2}} \right) \right] (e^1_\pm \hat{y} - e^2_\pm \hat{x}) \tag{28}
$$

3 Final Remarks

In conclusion, we have worked out the simplest solutions for the set of dynamical equations describing electromagnetic process in a Lorentz-symmetry breaking scenario taking into account QED non-linear induced effects, up to the first order in two parameters, in presence of an external constant magnetic field.

This has been used to reach a possibly useful expression to analyze the dispersion phenomena from astrophysical data coming from electromagnetic waves that have passed through a region where strong magnetic fields can be found, as in the surroundings of neutron stars or magnetars, since extreme precision can be reach using astrophysical sources [2,3,9]. In these regions even the bending of light by magnetic fields dominates the gravitational bending [21].

Peculiar polarization states for the photon field have also been found. It turns out that the photon acquires two distinct modes, each one with a single degree of freedom, when passing through a region of constant strong magnetic field in a Lorentz-violating background according to (27). By virtue of the breaking of Lorentz symmetry, the polarization modes indicate that the photon behaves as if it splits into independent scalars. Longitudinal polarization, like the characteristic $e^3_\pm$ in (27), has been object of interest in optics for a long time [22]. Because longitudinal components occur at the focal region of tightly focused laser beams a lot of optical techniques has been developed to study them, (see for instance [23,24] and references cited therein). In turn this can be useful to set upper bounds at laboratory scale of possible Lorentz-symmetry breaking models, in addition to the analysis of camouflage coefficients [25] that control this kind of violation in the SME.

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